Higgs bosons, t - quark, and the Nambu sum rule

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It may appear that the recently found resonance at $M_{H1} = 125$ GeV is not the only Higgs boson. We point out the possibility that the Higgs bosons appear in models of top - quark condensation, where the masses of the bosonic excitations are related to the top quark mass by the sum rule similar to the Nambu sum rule of the BCS - like theories. This rule was originally considered by Nambu in ^{3}He-B and in the BCS model of superconductivity. It relates the two masses of bosonic excitations existing in each channel to the fermion mass (example of the Nambu partners is provided by the amplitude and the phase modes in the BCS model describing Cooper pairing in the s-wave channel). We review these results and discuss the appearance of this rule in ^{3}He-A. Next, we consider how this rule appears in selected relativistic models including the models of top - quark condensation.

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I. INTRODUCTION

The recent experimental results [1, 2] on the 125 GeV Higgs exclude the appearance of the other Higgs bosons within the wide ranges of masses (approximately from 130 GeV to 550 GeV). However, this announced exclusion is related only to the particle with the same cross - section as the only standard Higgs boson of the Standard Model. The particles that have the smaller cross - sections are not excluded. To be more explicit, we refer to the recent data of CMS collaboration [3]. On Figure 4 the solid black curve separates the region, where the scalar particles are excluded (above the curve) from the region, where they are not excluded. For example, the particle with mass around 200 GeV and with the cross section about 1/3 of the Standard Model cross section is not excluded by these data. The similar exclusion curve was announced by ATLAS (plenary talk [4] at ICHEP 2012, slide 34).

The analogy with the superconductivity and superfluidity prompts that the Higgs boson may be composite. (See [5–7] for the foundation of the Higgs mechanism in quantum field theory.) Indeed, there were a lot of attempts to transform this idea into the model that does not contradict to experiment. Among such models the models of top quark condensation [8, 9] are of especial interest as they relate the Higgs boson to the only known Fermi particle (top - quark) with the mass of the same order as the Higgs boson mass.

In the theory of superconductivity that inspired the search for the composite model of Higgs boson there is an interesting relation between the scalar excitation mass and the mass of the fermion. This relation (which we call the Nambu sum rule) is also known in the superfluid ^{3}He-B. It was suggested by Nambu [10] and relates the values of the bosonic excitation masses in BCS - like theories with the value of the fermion mass, whose condensation leads to the formation of the condensate. In the form with one scalar mass it gives the relation between σ - meson mass and the constituent quark mass $M_\sigma \approx 2M_{quark}$ valid in the Nambu - Jona - Lasinio (NJL) approximation [11]. In the form $M_1^2 + M_2^2 = 4\Delta^2$ it is valid in the effective NJL - like model (i.e. the model with the 4 - fermion interaction) of ^{3}He-B for the boson masses $M_{1,2}$ existing at each value of $J = 0, 1, 2$, where J is the quantum number corresponding to the total angular momentum of the Cooper pair. It relates them to the constituent mass of the fermion excitation Δ existing due to the condensation (in the non-relativistic BCS theory the role of the masses of the fermionic and bosonic excitations is played by the energy gaps in the fermionic and bosonic spectrum respectively).

The analogy between the condensed matter models and the Higgs physics prompts that it would be instructive to investigate how the Nambu sum rule may appear (maybe, in a modified form) in the models of top quark condensation. It is worth mentioning that the traditional forms of the $t-\bar{t}$ pairings most likely are excluded by the present experimental data [8]. However, certain unconventional schemes may exist that give rise to several Higgs bosons satisfying the (modified) Nambu sum rule. We suggest to look for such schemes basing on the analogy with superfluid ^{3}He (we refer to the book [12] and to the references therein). For this

reason we review the bosonic excitations in both ^{3}He-B and ^{3}He-A. Next, we consider how the Nambu sum rule emerges in the relativistic models including the models of dense quark matter and the models of top - quark condensation. In the latter models, which we considered, the Nambu sum rule appears in the form

$$\sum M_{H,i}^2 \approx 4M_T^2, \tag{1}$$

where $M_T \approx 174$ GeV is the t - quark mass, and the sum is over the Higgs bosons existing within the given channel. In the other considered models the direct analogues of this rule appear. Sometimes the sum rule is violated. But in all considered cases (including ^{3}He-A) this occurs when the given channel contains the massless Goldstone bosons only.

According to Eq. (1) in the channel with only two Higgs bosons the Nambu partner of the recently found Higgs particle should have mass approximately equal to ≈ 325 GeV. It is worth mentioning that in 2011 the CDF collaboration [13] has announced the preliminary results on the excess of events in $ZZ \to ll\bar{l}\bar{l}$ channel at the invariant mass ≈ 325 GeV. CMS collaboration also reported a small excess in this region [14]. Although the particle with the cross sections of the Standard Model Higgs is excluded at this mass, this exclusion does not work for the particles with smaller cross sections. Originally the mentioned excess of events was treated as a statistical fluctuation. However, in [15, 16] it was argued that it may point out to the possible existence of a new scalar particle with mass $M_{H2} \approx 325$ GeV.

For two Higgs bosons with equal masses in one channel Eq. (1) predicts the value of mass around ≈ 245 GeV. Again, a certain excess of events in this region has been observed by ATLAS in 2011 (see, for example, [17]).

The paper is organized as follows. In Section 2 we review bosonic excitations and Nambu sum rules in ^{3}He. In section 3 we consider Nambu sum rules in the relativistic models. In section 4 we end with the conclusions.

II. NAMBU SUM RULES IN HELIUM - 3 SUPERFLUID

A. "Hydrodynamic action" in ^{3}He

According to [18] Helium - 3 may be described by the effective theory with the action

$$S = \int dt d^3x \bar{\chi}_s \{ i\partial_t + \mu + \frac{1}{2m}\Delta \} \chi_s - \frac{1}{2} \int dt \int d^3x \int d^3y u(x-y) \sum_{s,s'} \bar{\chi}_s(x,t)\chi_s(x,t)\bar{\chi}_{s'}(y,t)\chi_{s'}(y,t) \tag{2}$$

Here χ is anticommuting spinor variable, $s = \pm$, μ is the chemical potential, $u(x)$ is the interatomic potential. Then, the integration over the "fast" Fermi - fields (i.e., those with the sufficiently large values of momenta) gives the effective action for the modes living near to the Fermi surface. Assuming imaginary time and the spin-triplet p-wave pairing (i.e. the Cooper pairing in the state with orbital angular momentum $L = 1$ and spin angular momentum $S = 1$), in the first approximation this effective action can be written as

$$S_{low} = \sum_{p,s} \bar{a}_s(p)\epsilon(p)a_s(p) - \frac{g}{\beta V} \sum_{p,i=1,2,3} \bar{J}_i(p)J_i(p), \tag{3}$$

where

$$p = (\omega, k), \quad \hat{k} = \frac{k}{|k|}, \quad \epsilon(p) = Z^{-1}(i\omega - v_F(|k| - k_F))$$
$$J_i(p) = \frac{1}{2} \sum_{p_1+p_2=p} (\hat{k}_1 - \hat{k}_2) a_\alpha(p_2)\sigma_i a_\beta(p_1)\epsilon^{\alpha\beta} \tag{4}$$

Here $a_\pm(p)$ is the fermion variable in momentum space, v_F is Fermi velocity, k_F is Fermi momentum, g is the effective coupling constant. The authors of [18] proceed with the bosonization using the following trick. The unity is substituted into the functional integral that is represented as $1 \sim \int D\bar{c}Dc \exp(\frac{1}{g}\sum_{p,i,\alpha} \bar{c}_{i,\alpha}(p)c_{i,\alpha}(p))$, where $c_{i,\alpha}, (i, \alpha = 1, 2, 3)$ are bosonic variables. These variables may

be considered further as the field of the Cooper pairs, which serves as the analog of the Higgs field in relativistic theories. Shift of the integrand in $D\bar{c}Dc$ removes the 4 - fermion term. Therefore, the fermionic integral can be taken. As a result we arrive at the "hydrodynamic" action for the Higgs field c:

$$S_{eff} = \frac{1}{g}\sum_{p,i,\alpha}\bar{c}_{i,\alpha}(p)c_{i,\alpha}(p) + \frac{1}{2}\log \mathrm{Det} M(\bar{c},c), \tag{5}$$

where

$$M(\bar{c},c) = \left(\begin{array}{cc} Z^{-1}(i\omega - v_F(|k|-k_F))\delta_{p_1p_2} & \frac{1}{(\beta V)^{1/2}}[(n_1-n_2)c_\alpha(p_1+p_2)]\sigma_\alpha \\ -\frac{1}{(\beta V)^{1/2}}[(n_1-n_2)c_\alpha(p_1+p_2)]\sigma_\alpha & -Z^{-1}(i\omega - v_F(|k|-k_F))\delta_{p_1p_2} \end{array} \right) \tag{6}$$

B. Nambu sum rules in ^{3}He-B

In the B - phase of ^{3}He the condensate is formed in the state with $J=0$, where $\mathbf{J}=\mathbf{L}+\mathbf{S}$ is the total angular momentum of Cooper pair [12]

$$c^{(0)}_{i\alpha}(p) = (\beta V)^{1/2} C\,\delta_{p0}\delta_{i\alpha}\,. \tag{7}$$

This corresponds to the symmetry breaking scheme $G\to H$ with the symmetry of physical laws $G = SO_L(3)\times SO_S(3)\times U(1)$ and the symmetry of the degenerate vacuum states $H = SO_J(3)$. The parameter C satisfies the gap equation

$$0 = \frac{3}{g} - \frac{4Z^2}{\beta V}\sum_p(\omega^2 + v_F^2(|k|-k_F)^2 + 4C^2Z^2)^{-1} \tag{8}$$

The value $\Delta = 2CZ$ is the constituent mass of the fermion excitation. There are 18 modes of the fluctuations $\delta c_{i\alpha} = c_{i\alpha} - c^{(0)}_{i\alpha}$ around this condensate. Tensor $\delta c_{i\alpha}$ realizes the reducible representation of the $SO_J(3)$ symmetry group of the vacuum (acting on both spin and orbital indices). The mentioned modes are classified by the total angular momentum quantum number $J=0,1,2$.

According to [19, 20] the quadratic part of the effective action for the fluctuations around the condensate has the form:

$$S^{(1)}_{eff} = \frac{1}{g}(u,v)[1-g\Pi]\left(\begin{array}{c} u \\ v \end{array}\right), \tag{9}$$

where $\delta c_{i\alpha}(p) = u_{pi\alpha} + iv_{pi\alpha}$, and the polarization operator at $k=0$ is given by

$$\Pi = \left(\begin{array}{cc} \Pi^{uu} & 0 \\ 0 & \Pi^{vv} \end{array}\right) \tag{10}$$

At each value of $J=0,1,2$ the modes u and v are orthogonal to each other and correspond to different values of the bosonic energy gaps.

At $k=0$ the polarization operator can be represented as

$$\Pi(\omega) = \int_0^\infty dz \frac{\rho(z)}{z+\omega^2}, \tag{11}$$

where the spectral function $\rho \sim \sum |F_{Q\to ff}|^2$, and $|F_{Q\to ff}|^2$ is the probability that the given mode Q (in case of ^{3}He-B the quantum number $Q=J$) decays into two fermions.

At $J=0$, the v - bosonic mode is gapless that can easily be obtained using the gap equation. Also this follows from the fact that this is the Goldstone mode, which comes from the broken $U(1)$ symmetry. Next,

for any J we have ($\sqrt{t} = \epsilon_+ + \epsilon_-$; $k_+ = -k_-$; $\epsilon_\pm^2 - v_F^2(|k| - k_F)^2 - \Delta^2 = 0$):

$$
\begin{aligned}
\rho^u(t) &\sim \theta(t - 4\Delta^2)\sqrt{1 - \frac{4\Delta^2}{t}}\,\mathrm{Sp}\, G^{-1}(\epsilon_+, k_+)\, O_u^{(J)}\, G^{-1}(-\epsilon_-, k_-)\, O_u^{(J)} \\
&\sim \sqrt{1 - \frac{4\Delta^2}{t}}\,[(t/2 - \Delta^2) - \eta^{(J)}\Delta^2]\theta(t - 4\Delta^2) \\
\rho^v(t) &\sim \theta(t - 4\Delta^2)\sqrt{1 - \frac{4\Delta^2}{t}}\,\mathrm{Sp}\, G^{-1}(\epsilon_+, k_+)\, O_v^{(J)}\, G^{-1}(-\epsilon_-, k_-)\, O_v^{(J)} \\
&\sim \sqrt{1 - \frac{4\Delta^2}{t}}\,[(t/2 - \Delta^2) + \eta^{(J)}\Delta^2]\theta(t - 4\Delta^2) \qquad (12)
\end{aligned}
$$

Here

$$
\begin{aligned}
&G^{-1}(\epsilon, k) = \begin{pmatrix} Z^{-1}(\epsilon - v_F(|k| - k_F)) & 2C(\hat{k}\sigma) \\ -2C(\hat{k}\sigma) & Z^{-1}(\epsilon + v_F(|k| - k_F)) \end{pmatrix},\ O_{u,v}^{ij} = \begin{pmatrix} 0 & \hat{k}_+^i\sigma^j \\ \mp\hat{k}_+^i\sigma^j & 0 \end{pmatrix}, \\
&O^{(0)} = \frac{1}{\sqrt{D}}O^{ii}, \quad [O^{(1)}]^{ij} = \frac{1}{\sqrt{D(D-1)/2}}O^{[ij]}, \quad [O^{(2)}]^{ij} = \frac{1}{\sqrt{D(D+1)/2 - 1}}[O^{\{ij\}} - \frac{1}{D}O^{kk}\delta^{ij}], \qquad (13)
\end{aligned}
$$

with $D = 3$ and

$$
\eta^{(J)} = \frac{\mathrm{Sp}\, V O^{(J)} V O^{(J)}}{\mathrm{Sp}\, O^{(J)} O^{(J)}}\,. \qquad (14)
$$

with

$$
V = \begin{pmatrix} 0 & \hat{k}_+\sigma \\ -\hat{k}_+\sigma & 0 \end{pmatrix} \qquad (15)
$$

In the v - channel at $J = 0$ the energy gap is equal to zero that leads to the condition

$$
\mathrm{const} \int_{4\Delta^2}^{\Lambda^2} \sqrt{1 - \frac{4\Delta^2}{t}}\, dt = \frac{3}{g}, \qquad (16)
$$

where Λ is the ultraviolet cutoff. The bosonic energy gaps $E_{u,v}^{(J)}$ are defined by the equation:

$$
\mathrm{const} \int_{4\Delta^2}^{\Lambda^2} \sqrt{1 - \frac{4\Delta^2}{t}}\, \frac{t - 2\Delta^2(1 \pm \eta^{(J)})}{t - [E_{u,v}^{(J)}]^2}\, dt = \frac{3}{g} \qquad (17)
$$

with the same constant as in Eq. (16). Comparing these two equations we come to

$$
E_{u,v}^{(J)} = \sqrt{2\Delta^2(1 \pm \eta^{(J)})}\,, \qquad (18)
$$

which proves the Nambu sum rule for ^{3}He-B:

$$
[E_u^{(J)}]^2 + [E_v^{(J)}]^2 = 4\Delta^2 \qquad (19)
$$

Explicit calculation of (14) gives $\eta^{J=0} = \eta^{J=1} = 1$, and $\eta^{J=2} = \frac{1}{5}$. Thus we get immediately the result obtained in [20] via the direct solution of the equation $\mathrm{Det}\Big(g\Pi(iE) - 1\Big) = 0$:

1. $J = 0$.

   For $J = 0$ there is one pair of the Nambu partners (the gapless Goldstone sound mode and the so-called pair-breaking mode with the energy gap $E = 2\Delta$):

$$
E_1^{(0)} = 0, \quad E_2^{(0)} = 2\Delta \qquad (20)
$$

2. $J = 1$.

   For $J = 1$ there are three pairs of Nambu partners (three gapless Goldstone modes – spin waves and three corresponding pair-breaking modes with the energy gap $E = 2\Delta$):

$$E_1^{(1)} = 0, \quad E_2^{(1)} = 2\Delta \tag{21}$$

3. $J = 2$.

   For $J = 2$ there exist five pairs – five the so-called real squashing modes with the energy gap $E = \sqrt{2/5}\,(2\Delta)$ and correspondingly five imaginary squashing modes with the energy gap $E = \sqrt{3/5}\,(2\Delta)$:

$$E_1^{(2)} = \sqrt{2/5}\,(2\Delta), \quad E_2^{(2)} = \sqrt{3/5}\,(2\Delta)\,. \tag{22}$$

   (Zeeman splitting of imaginary squashing mode in magnetic field has been observed in [21], for the latest experiments see [22].)

C. Nambu sum rules in ^{3}He-A

1. *Anisotropic A-phase*

In the A - phase of ^{3}He the condensate is formed in the state with $S_z = 0$ and $L_z = 1$ [12]. In the orbital sector the symmetry breaking in ^{3}He-A is similar to that in the electroweak theory: $U(1) \otimes SO_L(3) \to U_Q(1)$, where the quantum number Q plays the role of the electric charge (see e.g. Ref. [23]), while in the spin sector one has $SO_S(3) \to SO_S(2)$. According to [24] one has

$$c^{(0)}_{i\alpha}(p) = (\beta V)^{1/2} C\, \delta_{p0}(\delta_{i1} + i\delta_{i2})\delta_{\alpha 3} = (\beta V)^{1/2} C\, \delta_{p0} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & i \\ 0 & 0 & 0 \end{pmatrix}\,, \tag{23}$$

where C satisfies the gap equation

$$0 = \frac{1}{g} - \frac{2Z^2}{\beta V} \sum_p \frac{1 - k_3^2}{\omega^2 + v_F^2(|k| - k_F)^2 + 4C^2 Z^2 (1 - k_3^2)}\,. \tag{24}$$

The A-phase is anisotropic. The special direction in the orbital space appears that is identified with the direction of the spontaneous orbital angular momentum of Cooper pairs, which is here chosen along the axis z. In this phase fermions are gapless. However, the value $\Delta(\theta) = 2CZ\sqrt{1 - k_3^2} = \Delta_0 \mathrm{sin}\theta$ may be considered as the technical gap depending on the direction in space that enters the expressions to be considered below. (Here θ is the angle between the anisotropy axis and the direction of the momentum k.)

In the BCS theory of ^{3}He-A, all bosonic modes are triply degenerate. This is the consequence of the hidden symmetry of the BCS theory applied to ^{3}He-A, which in particular gives rise to 9 gapless Goldstone modes instead of 5 modes required by symmetry breaking [25, 26]. On the language of $c_{i\alpha}$ this hidden symmetry leads to the representation of the one - loop effective action as the sum of the three terms. Each of that terms depends on $c_{i\alpha}$ with definite value of $\alpha = 1, 2, 3$. The term with c_{i3} is transformed into the term with c_{i2} via the substitution $c_{i3} \to ic_{i2}$. The term with c_{i2} is transformed into the term with c_{i1} via the substitution $c_{i2} \to c_{i1}$. Among 5 Goldstone bosons corresponding to the breakdown pattern $U(1) \otimes SO_L(3) \otimes SO_S(3) \to U_Q(1) \otimes SO_S(2)$ there are $u_{11} + v_{21}, u_{12} + v_{22}, u_{23} - v_{13}$ that are transformed to each other by the mentioned above transformation. Also there are the Goldstone modes u_{33}, v_{33}. The latter modes may be transformed by this transformation to $u_{31}, u_{32}, v_{31}, v_{32}$. Therefore, four additional gapless modes appear in weak coupling limit. Recall that in the strong coupling regime these four modes become gapped.

The values of the energy gaps are given by the solutions of the equation $\mathrm{Det}\Big(g\Pi(iE) - 1\Big) = 0$. Exact solutions of the given equations are presented in [24]. The energy gaps are complex - valued that means that the states are not stable. (The decay into the massless fermions is possible.) However, the real parts of

the energy gaps can be evaluated in the approximation, when the effective action at $k = 0$ is represented as the sum of the two terms: the first term corresponds to $\omega = 0$ while the second term is proportional to ω^2. Such a calculation gives the mass term for the modes of the field $c_{i\alpha}$ with the contribution due to the terms depending on higher powers of ω disregarded. This procedure gives six unpaired gapless Goldstone modes and two pairs of modes (triply degenerated) that satisfy a version of the Nambu sum rule. In this case the role of the square of the fermion mass is played by the angle average of the square of the anisotropic gap:

$$\bar{\Delta}^2 \equiv \left\langle \Delta^2(\theta) \right\rangle = \frac{2}{3}\Delta_0^2 \,. \tag{25}$$

The Nambu pairs are the following:

1. One (triply degenerated) pair of bosons (the phase and amplitude collective modes in Nambu terminology) is formed by the "electrically neutral" ($Q = 0$) massless Goldstone mode and the "Higgs boson" also with $Q = 0$:

$$E_1^{(Q=0)} = 0, \quad E_2^{(Q=0)} = 2\bar{\Delta} = \sqrt{8/3}\Delta_0 \tag{26}$$

2. The other (triply degenerated) pair represents the analog of the charged Higgs bosons in ^{3}He-A with $Q = \pm 2$ (see e.g. [26]). These are the so-called clapping modes whose energies are

$$E_1^{(Q=2)} = E_2^{(Q=-2)} = \sqrt{2}\bar{\Delta} = \sqrt{4/3}\Delta_0 \tag{27}$$

One can see that the spectrum of fermions and bosons in anisotropic superfluid ^{3}He-A also satisfies the Nambu conjecture written in the form

$$E_1^2 + E_2^2 = 4\bar{\Delta}^2 \tag{28}$$

(for each of the two pairs listed above) with the "average fermion gap" given by Eq. (25).

Alternatively these values may be obtained if in Eq. (2.16) of [24] the values of $\Delta^2(\theta)$ are substituted by their averages $\bar{\Delta}^2 \equiv \left\langle \Delta^2(\theta) \right\rangle = \frac{2}{3}\Delta_0^2$. Then, the integrals are omitted and we obtain the above listed values of the gaps.

As it was mentioned above, in the anisotropic systems in which the fermionic energy gap has zeroes, the spectrum of massive collective modes has imaginary part due to radiation of the gapless fermions. That is why the Nambu rule is not obeyed for the pole masses, but is obeyed for the mass parameters which are real, since they are determined at $\omega = 0$. In the systems, in which radiation is absent, such as isotropic fully gapped superfluid ^{3}He-B, the pole masses of the collective modes coincide with their mass parameters.

2. *Superfluid phases in 2+1 films*

The same relations (26) and (27) take place for the bosonic collective modes in the quasi two-dimensional superfluid ^{3}He films. There are two possible phases in thin films, the a-phase and the so-called planar phase (b phase in the terminology of Ref. [27]). Both phases have isotropic gap Δ in the 2D case, as distinct from the 3D case where such phases are anisotropic with zeroes in the gap.

We have the effective action for the bosonic degrees of freedom Eq. (5), Eq. (6) with the 2×3 matrix $c_{i\alpha}$. The following two forms of these matrices correspond to the a - and b - phases [27]:

$$\begin{aligned} c^{(0)}_{i\alpha}(p) &= (\beta V)^{1/2} C\, \delta_{p0} \begin{pmatrix} 1 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \; (a-phase) \\ c^{(0)}_{i\alpha}(p) &= (\beta V)^{1/2} C\, \delta_{p0} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \; (b-phase)\,. \end{aligned} \tag{29}$$

Let us consider the second possibility (the planar b - phase). We have the symmetry breaking pattern $SO(2) \otimes SO(3) \otimes U(1) \to SO(2)$. Correspondingly, there are four gapless Goldstone modes. Among them

there are u_{13} and u_{23} modes. Modes v_{13} and v_{23} are their partners with the energy gaps 2Δ. The analysis is similar to that of the s - wave superconductor.

As for the modes u_{ij}, v_{ij} with $i, j = 1, 2$, the analysis is similar to that of the ^{3}He-B phase. The spectral densities $\rho_{u,v}$ differ from those of Eq. (12) by the kinematic factor $\sqrt{1/t}$ instead of $\sqrt{1 - 4\Delta^2/t}$. Next, we substitute $D = 2$ to Eq. (13), and get

$$E^{(J)}_{u,v} = \sqrt{2\Delta^2(1 \pm \eta^{(J)})}\,, J = 0, 1, 2 \tag{30}$$

Direct calculation of (14) gives $\eta^{J=0} = -\eta^{J=1} = 1$, and $\eta^{J=2} = 0$. (In this case J is not the total momentum of the Cooper pair.) The resulting spectrum is

$$E^{(0)}_{u,v} = 2\Delta, 0;\ E^{(1)}_{u,v} = 0, 2\Delta;\ \text{and} \quad E^{(2)}_{1,u,v} = \sqrt{2}\Delta;\ E^{(2)}_{2,u,v} = \sqrt{2}\Delta\,. \tag{31}$$

In the a phase the symmetry breaking is $SO(2) \otimes SO(3) \otimes U(1) \to U(1)_Q \otimes SO(2)$ with three Goldstone modes. Acting as above, for the b - phase (in this case the u and v modes are mixed unlike the b - phase) or, applying the results of Ref. [27], one obtains that these modes form two pairs of Nambu partners (triply degenerated), with $Q = 0$ and $|Q| = 2$:

$$E^{(Q=0)}_1 = 0 \quad , \quad E^{(Q=0)}_2 = 2\Delta \quad \text{and} \quad E^{(Q=+2)} = \sqrt{2}\Delta \quad , \quad E^{(Q=-2)} = \sqrt{2}\Delta\,. \tag{32}$$

Note that since masses of $Q = +2$ and $Q = -2$ modes are equal, the Nambu sum rule necessarily leads to the definite value of the masses of the "charged" Higgs bosons.

It is worth mentioning that, in principle, the derivation of the energy gaps for the a phase with minor modifications may be applied also for the evaluation of the real parts of the energy gaps of the $3D$ A - phase. In such calculations dealing with the equations that are the analogues of Eq. (14) we need to substitute the angle averaged fermionic gap (25).

Because of the common symmetry breaking scheme in the electroweak theory and in ^{3}He-A we consider the listed above energy gaps as an indication of the existence of the Higgs boson with mass

$$M_H = \sqrt{2}M_T\,. \tag{33}$$

This mass is about 245 GeV, which is roughly twice the mass of the lowest energy Higgs boson.

III. NAMBU SUM RULES IN THE RELATIVISTIC MODELS OF TOP QUARK CONDENSATION

A. Effective action for bosonic modes

In this section we discuss the Nambu sum rule in the context of the extended NJL model of top - quark condensation. We consider here the toy model that generalizes the models of [9, 28, 29]. In addition to the third generation of quarks (t, b) two more fermions (U, D) are involved. These may be the quarks of the first or the second generation or new particles with the quantum numbers of up and down quarks. Anyway, we suppose that their masses are much smaller than M_T. Unlike the models of [9, 28, 29] our toy model includes the neutral current $t\bar{U}$. It was considered earlier in [30] but the bosonic spectrum was not analysed in this model. At the present moment we do not wish to define the realistic theory aimed to explain DEWSB and the formation of fermion masses. Our objective is to demonstrate how the Nambu sum rule (probably, in the modified form) may appear in the relativistic models of general kind.

The action has the form

$$S = \int d^4x \Big(\bar{\chi}[i\nabla\gamma]\chi + g(\bar{\chi}_{\alpha,L}\chi^{\beta}_R)[WW^+]^{\bar{\beta}}_{\beta}(\bar{\chi}_{\bar{\beta},R}\chi^{\bar{\alpha}}_L)[YY^+]^{\alpha}_{\bar{\alpha}}\Big) \tag{34}$$

Here $\chi^T_{\alpha,L} = (U, D)_L; (t, b)_L$ is the set of left - handed doublets with the generation index α; $\chi_{\beta,R} = U_R; t_R$ is the set of the (up) right - handed singlets with the generation index β. The matrices $W = \mathrm{diag}(w_U, w_t), Y = \mathrm{diag}(y_U, y_t)$ are supposed to be diagonal.

As in Section 2 we introduce the bosonic variable c^{β}_{α} and insert into the functional integral the expression $1 \sim \int D\bar{c}Dc \exp(\frac{1}{g}\sum_p \bar{c}^{\beta}_{\alpha}(p)c^{\alpha}_{\beta}(p))$. Shift of the integration variable eliminates the four - fermion term. After the integration over the fermion fields we arrive at the action

$$S_{eff} = -\frac{1}{g}\sum_p \bar{c}^{\beta}_{\alpha}(p)c^{\alpha}_{\beta}(p) + \log \mathrm{Det} M(\bar{c}, c), \tag{35}$$

where

$$\bar{\chi}_{p_1} M(\bar{c}, c)\chi_{p_2} = \bar{\chi}_{p_1}\hat{p}\gamma\chi_{p_2}\delta_{p_1p_2} - \frac{1}{(\beta V)^{1/2}}(W^{\bar{\beta}}_{\beta}Y^{\alpha}_{\bar{\alpha}}c^{\bar{\alpha}}_{\bar{\beta}}(p_1 - p_2)\bar{\chi}_{p_1,\alpha,L}\chi^{\beta}_{p_2,R} + h.c.) \tag{36}$$

We consider the situation when the condensate $\langle \bar{t}t\rangle$ is much larger than $\langle U\bar{U}\rangle$. Then

$$c^{(0)\beta}_{\alpha}(p) \approx (\beta V)^{1/2}\,\delta_{p0}\left(\begin{array}{cc} C_T & 0 \\ 0 & C_U \end{array}\right), \quad C_T >> C_U \tag{37}$$

The condensate gives the mass to the top quark $M_T \approx 2C_T$ and the mass of $M_U \approx 2C_U << M_T$. The gap equations read:

$$0 = \frac{1}{gN_C} - \frac{2w_q^2y_q^2}{\beta V}\sum_p (p^2 + M_q^2)^{-1}, \; q = U, t \tag{38}$$

Here $N_C = 3$ is the number of colors. It is implied here that $M_U << M_T$.

There are several modes of the fluctuations of c^{β}_{α} around this condensate. The quadratic part of the effective action for the fluctuations around the condensate has the form:

$$S^{(1)}_{eff} = \frac{1}{g}(u, v)[1 - gN_C\Pi]\left(\begin{array}{c} u \\ v \end{array}\right), \tag{39}$$

where $\delta c_{\alpha\beta}(p) = u_{p\alpha\beta} + iv_{p\alpha\beta}$. u - mode corresponds to the scalar while v - mode corresponds to the pseudoscalar. The polarization operator at $k = 0$ is given by

$$\Pi_{\alpha_1\alpha_2} = \delta_{\alpha_1\alpha_2}\left(\begin{array}{cc} \Pi^u_{\alpha_1} & 0 \\ 0 & \Pi^v_{\alpha_1} \end{array}\right) \tag{40}$$

As in Section 2 at $k = 0$ the polarization operator can be represented as

$$\Pi(\omega) = \int_0^{\infty} dz \frac{\rho(z)}{z + \omega^2}, \tag{41}$$

with the spectral function ρ.

B. Higgs bosons masses

In the scalar/pseudoscalar $t\bar{t}$ channel we have $((\sqrt{t}, 0) = p_+ + p_-;\, p^2_{\pm} = M^2_T)$:

$$\begin{aligned}
\rho^S_{t\bar{t}}(t) &= \frac{1}{8\pi^2}\theta(t - 4M^2_T)\sqrt{1 - \frac{4M^2_T}{t}}\,\mathrm{Sp}\,(\gamma p_- + M_T)(\gamma p_+ - M_T) = \frac{1}{4\pi^2}\sqrt{1 - \frac{4M^2_T}{t}}\,(t - 4M^2_T)\theta(t - 4M^2_T) \\
\rho^P_{t\bar{t}}(t) &= \frac{1}{8\pi^2}\theta(t - 4M^2_T)\sqrt{1 - \frac{4M^2_T}{t}}\,\mathrm{Sp}\, i\gamma^5(\gamma p_- + M_T)i\gamma^5(\gamma p_+ - M_T) = \frac{1}{4\pi^2}\sqrt{1 - \frac{4M^2_T}{t}}\, t\theta(t - 4M^2_T)
\end{aligned} \tag{42}$$

Integrals in Eq. (41) are ultraviolet divergent. The regularization may be introduced in such a way that the upper limit in each integral is substituted by the finite cutoff (that may depend on the channel). Next, the $(t\bar{t})$ condensate provides the symmetry breaking. There should be Goldstone bosons corresponding to the

broken symmetry. This provides P excitation in $t\bar{t}$ channel is massless (the corresponding bilinear appears via the application of the generator of the broken symmetry to $(t\bar{t})$). Then, we have $\Pi^P_{t\bar{t}}(0) = \Pi^S_{t\bar{t}}(2iM_T)$ that means that the massive scalar excitation appears with mass $2M_T$. The same result can be obtained in the neutral channels $t\bar{t}, U\bar{U}$ via the direct calculation of the polarization operator:

$$
\begin{aligned}
\frac{1}{gN_C} - \Pi^S_{q\bar{q}}(iE) &= \frac{1}{gN_C} - \frac{iw_q^2 y_q^2}{2(2\pi)^4}\int d^4l\,\mathrm{Sp}\,\frac{1}{l\gamma - M_q}\,\frac{1}{(p-l)\gamma - M_q} \\
&= (p^2 - 4M_q^2)w_q^2 y_q^2\, I(M_q, M_q, p) \\
\frac{1}{gN_C} - \Pi^P_{q\bar{q}}(iE) &= \frac{1}{gN_C} - \frac{iw_q^2 y_q^2}{2(2\pi)^4}\int d^4l\,\mathrm{Sp}\, i\gamma^5\frac{1}{l\gamma - M_q}\, i\gamma^5\frac{1}{(p-l)\gamma - M_q} \\
&= (p^2)w_q^2 y_q^2\, I(M_q, M_q, p), \\
& I(m_1, m_2, p) = \frac{i}{(2\pi)^4}\int d^4l\,\frac{1}{(l^2 - m_1^2)[(p-l)^2 - m_2^2]}
\end{aligned}
\tag{43}
$$

Here gap equation (38) is used. We get

$$
M^P_{q\bar{q}} = 0; \quad M^S_{q\bar{q}} = 2M_q \tag{44}
$$

for $q = t, U$.

In the neutral channel that includes only one t - quark $t\bar{U}$ we have two scalar states (doubly degenerated, $M^P_{t\bar{U},i} = M^S_{t\bar{U},i}, i = 1,2$):

$$
\begin{aligned}
\frac{1}{gN_C} - \Pi_{t\bar{q},i}(iE) &= \frac{1}{gN_C} - \frac{iw_t^2 y_U^2}{2(2\pi)^4}\int d^4l\,\mathrm{Sp}\,\frac{1}{l\gamma - M_T}\,\frac{1}{(p-l)\gamma - M_U} \\
&\approx (p^2 - M_T^2)w_t^2 y_U^2\, I(M_T, M_U, p) + a\Big(\frac{1}{gN_C} - \frac{2iw_t^2 y_t^2}{(2\pi)^4}\int d^4l\,\frac{1}{l^2 - M_T^2}\Big) \\
&+ b\Big(\frac{1}{gN_C} - \frac{2iw_U^2 y_U^2}{(2\pi)^4}\int d^4l\,\frac{1}{l^2 - M_U^2}\Big) + w_t^2 y_U^2 \lambda J(M_T, M_U) \\
\frac{1}{gN_C} - \Pi_{t\bar{q},2}(iE) &= \frac{1}{gN_C} - \frac{iw_U^2 y_t^2}{2(2\pi)^4}\int d^4l\,\mathrm{Sp}\,\frac{1}{l\gamma - M_T}\,\frac{1}{(p-l)\gamma - M_U} \\
&\approx (p^2 - M_T^2)w_U^2 y_t^2\, I(M_T, M_U, p) + \bar{b}\Big(\frac{1}{gN_C} - \frac{2iw_t^2 y_t^2}{(2\pi)^4}\int d^4l\,\frac{1}{l^2 - M_T^2}\Big) \\
&+ \bar{a}\Big(\frac{1}{gN_C} - \frac{2iw_U^2 y_U^2}{(2\pi)^4}\int d^4l\,\frac{1}{l^2 - M_U^2}\Big) - w_U^2 y_t^2 \bar{\lambda} J(M_T, M_U), \\
& J(m_1, m_2) = \frac{i}{(2\pi)^4}\int d^4l\,[\frac{1}{l^2 - m_1^2} - \frac{1}{l^2 - m_2^2}]
\end{aligned}
\tag{45}
$$

In these expressions we introduced the following constants:

$$
\begin{aligned}
a &= \frac{\frac{w_U^2}{w_t^2} - 1}{\frac{w_U^2}{w_t^2} - \frac{y_t^2}{y_U^2}}, \quad b = \frac{\frac{y_t^2}{y_U^2} - 1}{\frac{y_t^2}{y_U^2} - \frac{w_U^2}{w_t^2}}, \quad \lambda = \frac{\frac{y_t^2}{y_U^2} + \frac{w_U^2}{w_t^2} - 2\frac{w_U^2}{w_t^2}\frac{y_t^2}{y_U^2}}{\frac{y_t^2}{y_U^2} - \frac{w_U^2}{w_t^2}}, \\
\bar{a} &= \frac{\frac{w_t^2}{w_U^2} - 1}{\frac{w_t^2}{w_U^2} - \frac{y_U^2}{y_t^2}}, \quad \bar{b} = \frac{\frac{y_U^2}{y_t^2} - 1}{\frac{y_U^2}{y_t^2} - \frac{w_t^2}{w_U^2}}, \quad \bar{\lambda} = -\frac{\frac{y_t^2}{y_U^2} + \frac{w_U^2}{w_t^2} - 2}{\frac{y_t^2}{y_U^2} - \frac{w_U^2}{w_t^2}}
\end{aligned}
\tag{46}
$$

These constants satisfy the following equations

$$
\begin{aligned}
& a + b = \bar{a} + \bar{b} = 1, \quad (1 - 2b\frac{w_U^2}{w_t^2}) + (1 - 2a\frac{y_t^2}{y_U^2})) = (1 - 2\bar{b}\frac{w_t^2}{w_U^2}) + (1 - 2\bar{a}\frac{y_U^2}{y_t^2})) = 0, \\
& \lambda = (-1 + 2a\frac{y_t^2}{y_U^2}), \quad \bar{\lambda} = (-1 + 2\bar{a}\frac{y_U^2}{y_t^2})
\end{aligned}
\tag{47}
$$

At $\Lambda >> M_T >> M_U$ we get $J(M_T, M_U) \approx M_T^2 I(M_T, M_U, p) \approx \frac{M_T^2}{16\pi^2} \log\Lambda^2/M_T^2$ (here Λ is the ultraviolet cutoff of the NJL model). Therefore we get

$$M^2_{t\bar{U},1} = M_T^2(1-\lambda); \quad M^2_{t\bar{U},2} = M_T^2(1+\bar{\lambda})) \tag{48}$$

The modified Nambu sum rule for $t\bar{U}$ channel has the form:

$$\sum_{i=1,2}\Big([M^P_{t\bar{U},i}]^2 + [M^S_{t\bar{U},i}]^2\Big) \approx 4M_T^2(1+\frac{\bar{\lambda}-\lambda}{2}) \approx 4M_T^2(1-\frac{y_t^2 w_t^2 + y_U^2 w_U^2 - w_t^2 y_U^2 - y_t^2 w_U^2}{y_t^2 w_t^2 - y_U^2 w_U^2}) \tag{49}$$

In $t\bar{t}$ channel we have the Nambu sum rule in its usual form, while in the $U\bar{U}$ channel the Nambu sum rule works with M_U instead of M_T. In addition three massless charged Higgs bosons appear in the channels $t\bar{D}, U\bar{D}, t\bar{b}$.

C. Particular cases

Since we have the modified Nambu Sum rule in the $t\bar{U}$ channel, the following particular cases are of interest:

1. $Y = 1$

   In this case we have (in the $t\bar{U}$ channel):

$$M^2_{t\bar{U},1} = 0; \quad M^2_{t\bar{U},2} = 2M_T^2 \tag{50}$$

   The modified Nambu sum rule for $t\bar{U}$ channel has the form:

$$\sum_{i=1,2}\Big([M^P_{t\bar{U},i}]^2 + [M^S_{t\bar{U},i}]^2\Big) \approx 4M_T^2 \tag{51}$$

2. $W = 1$

   In this case we have

$$M^2_{t\bar{U},2} = 0; \quad M^2_{t\bar{U},1} = 2M_T^2 \tag{52}$$

   Again, the modified Nambu sum rule has the form of Eq. (51).

3. $Y = W$

   We get

$$M^2_{t\bar{q},1} = M_T^2(1-\frac{y_t^2 - y_U^2}{y_t^2 + y_U^2}); \quad M^2_{t\bar{q},2} = M_T^2(1-\frac{y_t^2 - y_U^2}{y_t^2 + y_U^2}) \tag{53}$$

   Remind that the gap equations Eq. (38) have the form [9]

$$\frac{1}{gN_C} - \frac{y_U^2 w_U^2}{8\pi^2}(\Lambda^2 - M_U^2 \log\frac{\Lambda^2}{M_U^2}) = 0,\ q = t, U \tag{54}$$

   From here it follows that at $\Lambda^2 >> M_T^2 >> M_U^2$ the fine tuning is needed, and $|y_t - y_U| << 1$. That's why we get

$$M^2_{t\bar{U},1} \approx M_T^2; \quad M^2_{t\bar{U},2} \approx M_T^2, \tag{55}$$

   Again, the sum rule in the form of Eq. (51) holds.

4. $|y_t - y_U| << 1$, $|w_t - w_U| << 1$
From the consideration of the gap equations (see the previous case) it follows that $|y_t^2 w_t^2 - y_U^2 w_U^2| << 1$. We consider here the case, when $y_t^2 - y_U^2 = A\epsilon$, $w_t^2 - w_U^2 = B\epsilon$, $\epsilon << 1$. Then

$$\sum_{i=1,2}\Big([M^P_{t\bar{q},i}]^2 + [M^S_{t\bar{q},i}]^2\Big) \approx 4M_T^2\Big(1 - \frac{y_t^2 w_t^2 + y_U^2 w_U^2 - w_t^2 y_U^2 - y_t^2 w_U^2}{y_t^2 w_t^2 - y_U^2 w_U^2}\Big) \approx 4M_T^2\Big(1 - \frac{AB}{A w_U^2 + B y_U^2}\epsilon\Big) \approx 4M_T^2 \tag{56}$$

Thus we come again to Eq. (51).

Since $|y_t^2 w_t^2 - y_U^2 w_U^2| << 1$ is needed in order for the finite values of masses are obtained from the gap equation, it would be natural if both $|y_t - y_U| << 1$ and $|w_t - w_U| << 1$. We consider the other cases as marginal. So, we come to the following

Lemma 1. *In non - marginal cases considered above, i.e. when $y_t \sim y_U; w_t \sim w_U$, the Nambu sum rule in the form of Eq. (1) holds both in the $t\bar{U}$, and $t\bar{t}$ channels.*

It is worth mentioning that the simplest relativistic models of the kind discussed in this section were considered in [9, 11, 28]. In these models the neutral Higgs bosons have masses 0 or $2M_T$. However, the model considered in [28] has the charged Higgs bosons with masses $\approx \sqrt{2}M_T$. Actually, our derivation of the masses in $t\bar{q}$ channels is similar to that of [28] for the charged Higgs bosons.

A further generalization of the model of [28] was considered in [29], where three scalar Higgs doublets are to be introduced, the fourth generation of quarks with large masses is involved. In this model there are two charged scalar Higgs modes with masses $M_1^{H^\pm}, M_2^{H^\pm}$, and two pseudoscalar modes with masses M_1^A, M_2^A that satisfy the following relation $2\sum_i\Big([M_i^{H^\pm}]^2 - [M_i^A]^2\Big) \approx 4M_T^2$.

D. Nambu sum rules in dense QCD

Among the other relativistic systems, where the analogues of the Nambu sum rules were observed, we would like to mention QCD in the presence of finite chemical potential. First, let us notice the normal phase with the broken chiral symmetry (both T and μ are small compared to the QCD scale Λ_{QCD}). We already mentioned in the introduction, that in this phase the NJL approximation leads to the Nambu sum rule in the trivial form $M_\sigma = 2M_{quark}$. However, at nonzero $\mu << M_{quark}$ the Nambu sum rule in the nontrivial form appears for the diquark states. Namely, the following values of the masses of the diquarks are presented in Eq. (46) of [31]:

$$M^2_\Delta = (2M_{quark} - 2\mu)^2; \quad M^2_{\Delta^*} = (2M_{quark} + 2\mu)^2 \tag{57}$$

So that

$$M^2_\Delta + M^2_{\Delta^*} \approx 2 \times 4M^2_{quark}\,\mathrm{at}\,\mu << M_{quark} \tag{58}$$

(Here Δ is the diquark while Δ^* is the antidiquark.)

In dense QCD with $\mu > \Lambda_{QCD}$ there may appear several phases with different diquark condensates. Among them there is, for example, the color - flavor locking phase (CFL). In the phenomenological models of this phase the three quarks u, d, s are supposed to be massless. The condensate is formed [32, 33]

$$\langle [\psi^i_\alpha]^t i\gamma^2\gamma^0\gamma^5\psi^j_\beta\rangle \sim \Phi^I_J\,\epsilon_{\alpha\beta J}\epsilon^{ijI} \sim (\beta V)^{1/2} C\,\epsilon_{\alpha\beta I}\epsilon^{ijI} \tag{59}$$

There are 18 scalar fluctuations of Φ around this condensate (there are also 18 pseudo - scalar fluctuations with the same masses [31]). The symmetry breaking pattern is $SU(3)_L \otimes SU(3)_R \otimes SU(3)_F \otimes U(1)_A \otimes U(1)_B \to SU(3)_{CF}$. That's why there are $9 + 9$ massless Goldstone modes. Among the remaining $9 + 9$ Higgs modes there are two octets of the traceless modes and two singlet trace modes. Correspondingly, the quark excitations also form singlets and octets. The singlet fermionic gap Δ_1 is twice larger than the octet fermionic gap Δ_8 (see Sect. 5.1.2. of [33]). Applying the technique similar to that of we developed for the consideration of ^{3}He-B we get the scalar singlet and octet masses $M_1 = 2\Delta_1, M_8 = 2\Delta_8$. This may also be derived from the results presented in [34, 35]. Thus, for the CFL phase of the color superconductor we have the Nambu sum rules in the trivial form.

IV. CONCLUSIONS AND DISCUSSION

In this paper we consider the possibility that there exist several Higgs particles with masses that satisfy the Nambu sum rule [10]. According to this sum rule the sum of the Higgs masses squared is equal to $2M_T$ squared ($M_T \approx 174$ GeV is the t - quark mass) in each channel. Originally this sum rule was considered by Nambu in ^{3}He-B and the conventional superconductivity. In the present paper we consider how the Nambu sum rule emerges in ^{3}He-A including the thin films. We also mention the analogue of this sum rule in QCD at finite chemical potential. Next, we consider the relativistic topcolor models. We consider the toy model that involves the third generation of quarks and the two additional fermions U, D that may be identified either with u, d or with c, s. Also the situation may be considered, in principle, when U and D are the new fermions with the quantum numbers of up and down quarks and with the masses smaller than the mass of the t - quark. We do not intend to consider this model as realistic. Our aim was to demonstrate how the sum rule Eq. (1) emerges in relativistic models. However, in principle, one may try to update this model in order to move it towards a realistic theory. Namely, there are two extra charged Goldstone bosons and one extra neutral Goldstone boson. Some physics is to be added in order to make them massive. The model contains the current $\bar{U}t$. The energy scale of the theory that has the considered NJL model as an approximation should be essentially larger than 1 TeV in order to pass the existing constraints on the FCNC that require $[1/g]^{1/2} \geq 10^3$ TeV [41]. This implies that $\Lambda \geq 10^3$TeV. The large value of Λ is also needed in order to provide the realistic value of $F_T \approx 245$ GeV. Let us remind that the considered NJL model requires the fine tuning of the parameters $w_{U,t}, y_{U,t}$ at $\Lambda >> M_T >> M_U$. This fine tuning provides both finite values of M_T, M_U and the validity of the Nambu sum rule Eq. (1). (When the ratio Λ/M_T is decreased, Eq. (1) becomes more and more approximate. In QCD the Nambu sum rule $m_\sigma \approx 2m_{quark}$ is not precise for the similar reason.) That's why even if the realistic theory is constructed on the basis of our toy model, it would be unnatural, i.e. rely on the fine tuning.

However, a different model, that inherits a structure of the considered toy model may be realistic. And the Nambu sum rule in this hypothetical model may appear in the form of Eq. (1). If this sum rule holds, and there are two (doubly degenerated) Higgs bosons in the channel that contains the 125 GeV Higgs, then the partner of the 125 GeV boson should have mass around 210 GeV. If there are only two states in this channel, then the partner of the 125 GeV Higgs should have the mass around 325 GeV. Then the two Higgs masses $M_{H1} = 125$ GeV and $M_{H1} = 325$ GeV satisfy the relations $M_{H1} = \sqrt{1/8}\,(2M_T), \quad M_{H2} = \sqrt{7/8}\,(2M_T)$. These relations are to be compared with Eq. (22). In the channel with two states of equal masses the 245 GeV Higgs bosons should appear in analogy with ^{3}He-A considered in Section 2. An important consequence of the Nambu sum rule (if it holds) is that the 125 GeV Higgs cannot be the partner of the neutral Goldstone boson eaten by the Z boson. According to Eq. (1) its partner should have mass around 350 GeV.

Still we do not have the general proof of the Nambu sum rule Eq. (1) in the BCS - like theories of general type. However, in all considered (non - marginal) cases it holds with only exceptions related to the channels with massless excitations only. It is worth mentioning, however, that these results belong to the NJL - like models considered in weak coupling. In any realistic models (like QCD, technocolor, topcolor, etc) this is only an approximation. However, as Nambu noticed in [10] this sum rule may work better than the NJL approximation itself.

We did not consider in this paper the possibility that the order parameter has the structure of space - time tensor as in ^{3}He (see e.g. [45–49]). The simplest models of this kind appear as a modification of our toy model with the action Eq. (34), where $\bar{\chi}_\alpha(p_+)O_{ij}\chi^\beta(p_-)$ stands instead of $\bar{\chi}_{\alpha,L}(p_+)\chi^\beta_R(p_-)$. Here O_{ij} is the space - time tensor composed of gamma - matrices and momenta $p_\pm$ [50]. There is the interesting possibility that within the models of such kind several condensates are formed. For example, in addition to the t - quark condensate the condensate of the form $\bar{\chi}i\gamma^{\{i}\nabla^{j\}}\chi \sim \Delta\delta^{ij}$ may appear. There are the spin 2 excitations of the order parameter $h^{ij} = \bar{\chi}i\gamma^{\{i}\nabla^{j\}}\chi$ within such a model. Under certain circumstances these fluctuations may become the gravitons of emergent quantum gravity.

It is also worth mentioning that Eq. (1) may be related to the condition for vanishing of the quadratic divergences in the vacuum energy in the absence of massive gauge fields [36–40].

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